

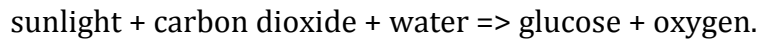


Northwest Math Sightings – Bracken Math

Students in our math classes legitimately ask us sometimes, “When will I ever need to know this stuff?” It’s a question that has many answers depending on who has asked the question and why. Over the years, good teachers develop a skill at fishing out the response that will work for this or that student at this or that moment. Sometimes it concerns an application in “real life,” sometimes it has to do with requirements for the next course down the curricular line, or for tests that must be passed and so forth. My favorite sort of answer, though, begins this way: “When you understand this math your life will be more interesting. Let me explain ...”

Spring arrives every year in the northwest and brings an awakening of plants that have lain dormant throughout the long grey fall and winter months. The earth moves around our star and because of the twenty three and a half degree tilt in the earth’s axis the northern hemisphere leans toward the sun and receives its rays more directly. While the winter sun sits low in the sky and its light is smeared out across the land, the higher sun angles of summer result in a greater intensity of the light – more photons per square inch - and plants respond sending out shoots and leaves to harvest the light.

This harvest of the sun’s light is crucial. Plants are fueled largely by a simple sugar called glucose and that substance is the result of photosynthesis, one of the most fundamental biological processes on our planet. A simplified representation of photosynthesis looks like this:



Now here is an amazing fact: each year terrestrial and marine plants make enough glucose to fill a freight train 30 million miles long (Hoagland, 1995). That’s enough to circle the earth more than 1000 times! Every year! The process requires sunlight and plants have evolved an array of strategies and structures that enable them to gather this vital resource.

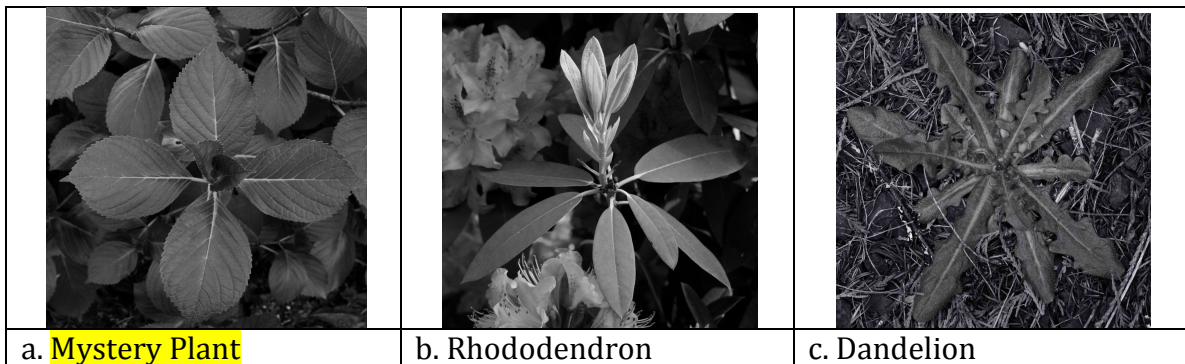


Figure 1

I am certainly not a botanist but even to my untrained eye the geometry and in particular, the symmetry is striking. Leaves radiate out from the central stem, filling the plane to different degrees and receiving varying amounts of sunlight as a result. In figure 1a, the plant sends out four broad leaves all at right angles to one another. In figure 1b the Rhododendron uses six but the leaves are slim and fill less of the space in this example. The low-lying dandelion in figure 1c has more than a dozen leaves some of which lay on top of others. Still in each case there is plenty of room between the leaves and sunlight streams through.

In this article we will concentrate on one plant, the Western Bracken Fern (*Pteridium aquilinum*), and the fascinating strategy it employs to gather sunlight. Along the way we will find good uses for mathematics in developing a more complete understanding of how this happens.



Figure 2 Bracken fern - side view

In Figure 2 you see an example of a Bracken fern found near Shelton, Washington. These plants are not hard to find; they grow in open areas, often along roadsides and across a wide range of climate zones throughout North America and around the world.



Figure 3 Bracken fern - top view

Figure 3 shows a Bracken fern from above. Take a close look at the two photos and consider the way in which the structure of the plant enables it to gather sunlight. What do you notice about the way in which the plant rises from the ground in Figure 2? What about the way the “leaves” sometimes overlap in Figure 3? How might you begin to apply some measurement and mathematics in order to really understand these things more fully? I

encourage you to take a few minutes to think about these things before you read on.

Before I go on let me say that while there are proper scientific names for the parts of the fern, (stalk, petiole, frond, pinnule, and so forth) I am not going to use them because that is not the point of this analysis. While I consider it good to call things by their proper names I want to get straight to the math. Perhaps more importantly, in a classroom I would not want to insist on the use of scientific nomenclature if I thought it would discourage students from diving in with the application of mathematics. Proper names should be introduced when they are needed. With that said, let's take a mathematical look at the fern.

The "trunk" of the fern makes approximately a 90° angle with the ground (assuming the ground is flat). This makes sense as it affords the fern the best chance to rise above the other plants and get at the sunlight. At this angle, every inch of growth puts the fern an inch above other plants that might shade it out. The first branches come from the trunk while it is still in this vertical mode but soon thereafter the trunk begins to bend and by the third or fourth branches the plant is often bent over 90 degrees or more, running parallel to the ground. The leaves, at first quite large, get smaller and smaller and the trunk becomes thinner as the whole plant narrows to its triangular finish. If you look carefully at Figure 2 you will see that the first set of branches and their leaves are over-lapped by those above them. You can see it even more clearly in Figure 3; moving from the bottom of the figure toward the top, the lower set of leaves is overlapped by the one above and beyond it as you move along the trunk and that set of leaves is overlapped by the next one, but to a lesser extent. In fact, looking farther along the trunk, the branches are found closer to one another but the leaves overlap less and less as you proceed to the end.

This constitutes a sort of pattern. I have looked at many Bracken ferns (too many, I've been told, but that's another story.) and I can tell you that this is a consistent pattern. Your students could take a look at some ferns if some are handy (Call it homework.) or do an image search if they are not but the Internet is.

Can you think of a way to use mathematics to make quantitative sense of this change in the degree of overlapping as you move along the trunk? The Common Core Standards are directing us (as the NCTM and Washington State standards did before them) to enable our students to pay attention to patterns and to increase their ability and inclination to use math to make sense of the world. This is a word problem that the world has written for us!

Here's what I did: I visited ten Bracken ferns and spent about 15 minutes with each one measuring first the height at which each of the first seven branches emerged from the trunk. I made a column on a sheet of notebook paper and labeled it "up." The "up" number is simply the height, the elevation at which the branch first leaves the trunk measured up from a spot directly beneath the junction on the trunk. Next I measured the distance from this point on the ground back to the place where the trunk leaves the ground. This became my "over" number. Taken together these two

numbers tell me how far up and how far over each junction is from the base of the fern. They are essentially the coordinates of the junction in some two-dimensional representation of the plant, where the origin is the point where the trunk emerges from the ground.

Next I measured the distance between the braches progressing from the first set to the second, from the second to the third and the so on moving out along the trunk as far as the 7th set of branches. Finally I measured the length of the leaves leaving the braches. I did this for the first full-sized leaves emitted from each of the first seven sets of branches. In Figure 5 you can see these distances identified.

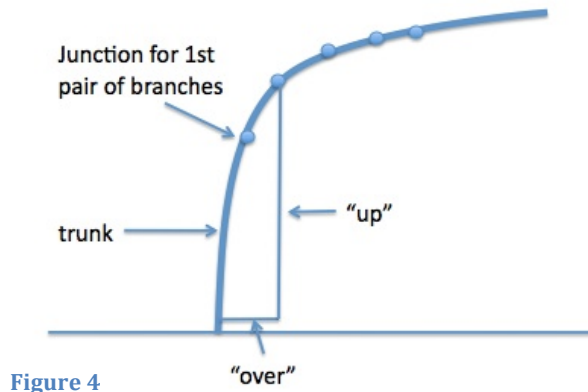


Figure 4



Figure 5 - Leaf lengths and distances between braches

I should be clear that this is not all I did. I started out looking at a variety of other factors, such as the angle at which the branches emerged from the trunk, the lengths of the branches, the complexity of the leaves and leaflets and so forth. This “messaging about” led me to understand what I wanted to look at. I noticed in this process that there was no end of other factors that might have been fascinating to explore but that they were more or less interesting to me according to my grasp of the mathematics needed to pursue the investigation. This is true for our students as well and points to the importance of differentiation in our curriculum.

If this has made sense, you can see that now I am in a position to calculate what I called the “overlap factor.” This is the degree to which the leaves coming from succeeding branches overlapped one another. When the sum of the lengths of the leaves from two successive braches is greater than the distance between the branches, then the ratio of this sum and the distance is greater than one. An overlap factor of two, for example results when the leaves are twice as long, taken together, as the gap between successive branches. An overlap factor less than one means that the leaves are not long enough to reach each other, to fill the space between successive branches. You will also see that with my “up” and “over” figures I can calculate an approximation to the slope of the trunk at the points where the 1st through 6th junctions occurred.

When all of the numbers have been crunched and calculated we end up with the results seen in Table 1. I have to say that it was a lot more fun to do the measurements and the calculations than it is to look at the finished product, boiled down to these few figures. It’s a little like cross country races – you get a lot more from running one than you do from standing at the finish line and watching the runners come in.

	1 st branch	2 nd branch	3 rd branch	4 th branch	5 th branch	6 th branch
Mean slope	5.6	1.5	0.8	0.5	0.4	0.2
	1 st – 2 nd	2 nd – 3 rd	3 rd – 4 th	4 th – 5 th	5 th – 6 th	6 th = 7 th
Overlap factor	1.7	1.5	1.4	1.2	1.1	0.9

Table 1

Nevertheless, we can see from the numbers that as we go up and then out along the curving trunk, from the first branches to the last, the slope, at first great, becomes more and more shallow and the overlap of the leaves becomes less and less pronounced until finally they do not, on average, meet one another. This makes sense because when the slope is large the trunk is still shooting upward and the leaves on successive sets of branches are far enough above one another that sunlight will get through. Whereas by the time the slope is nearing zero the overlap factor is near one – no overlap. This is good for the fern because any overlap would result in one leaf laying right atop and shading out the other. In the race to secure the sun’s energy and produce the sugars needed to build and grow, the Bracken fern has developed a strategy that seems to work.

What we as math teachers need is a teaching strategy that enables our students to harvest the energy that mathematics can bring to their lives. I do believe that being able to use math to make sense of the world is the long-term goal and when we get there our students will understand why this math is worth knowing.

References:

Hoagland, M. & Dodson, B. (1995). *The Way Life Works*. Random House Times Books, a division of Random House, Inc.

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