Northwest Math Sightings – Cooling Math

Students in our math classes legitimately ask us sometimes, “When will I ever need to know this stuff?” It’s a question that has many answers depending on who has asked the question and why. Over the years good teachers develop a skill at fishing out the response that will work for this or that student at this or that moment. Sometimes it concerns an application in “real life,” sometimes it has to do with requirements for the next course down the curricular line, or for tests the student must pass and so forth. My favorite answer, though, is this: “When you understand this math your life will be more interesting. Let me explain…”

When I order a latte or mocha or what-have-you at the local coffee place I am apt to add to my order, the request, ”Extra hot, please.” If I don’t, I find that I am often disappointed because for me a large part of what that drink needs to be is just hot. In this post-McDonalds-scalding-suit era, the temperature at which drinks are served and sold seems to have dropped considerably. I am interested therefore in ways to keep drinks hot and by extension, in the ways in which they cool down.

What *is* hot? My wife drinks tea and in order to make a good pot of that, she tells me, the pot must be pre-warmed and the water should be boiling when it is poured over the tea bag and into the pot. The pot is then insulated with a “tea cozy” (Do a image search for it on the Web and prepare yourself for a lot of purple and pink yarn.). It then has to steep for at least five minutes. If a cup is taken at this point the result is a very hot (88ºC) drink. My coffee maker does not keep the brewed coffee quite so hot, maintaining a temperature of about 75ºC, which is equivalent to about 165ºF (75x1.8+32=167ºF). That, for me, is hot enough; not so hot that it will scald the inside of my mouth but hot enough to be enjoyable. The problem is that it cools down. The kinetic energy of the molecules of liquid is transferred to the cup and to the air. The cup radiates energy and also passes kinetic energy to the air molecules that surround the cup. Those molecules are soon wafted away, carrying the heat with them. The cup of coffee cools and at some point the stuff is no longer hot enough to be enjoyable. Over time it will reach equilibrium with its surroundings and who wants room temperature coffee? I accept the notion that all good things come to an end and the cooling of the cup is a more or less inevitable part of life. Nevertheless, there are things we can do to understand and affect the process. A little mathematics and a little technology enable us to spend a few enjoyable hours investigating the situation.

Two Cups A couple of simple USB temperature probes cabled to my computer (or to a graphing calculator) enable me to gather data and see it graphed in real time. Figure 1 shows the graphs of temperature versus time for two cups of coffee. The first consisted of 10 ounces left to cool in a ceramic mug. The second was a similar amount in a similar cup from which I drank periodically until the cup was empty. The horizontal scale represents time and runs from zero to sixty minutes. The vertical axis is for temperature (degrees C) and runs from 15 to 75 degrees with divisions at 30ºC, 50ºC and 70ºC.

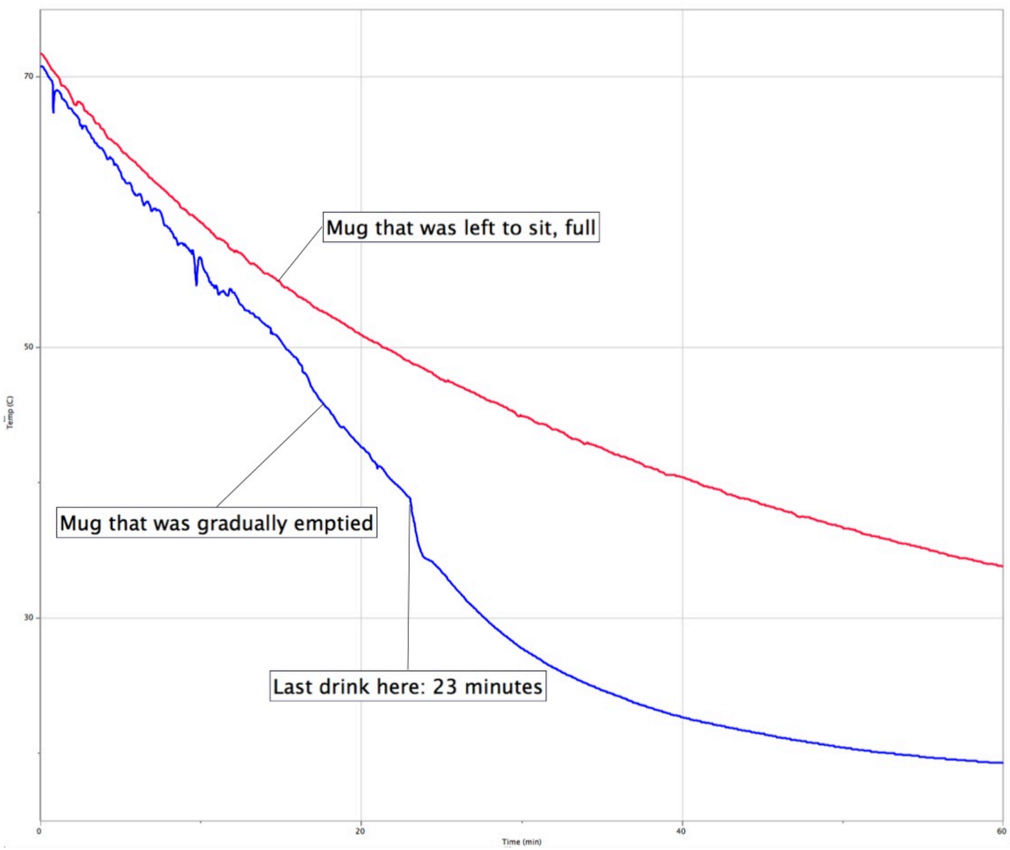

Figure 1. Cooling curves for two cups of coffee

I can analyze the data, looking for meaning and connections in the numbers and in the shapes of the graphs. A simple visual perusal reveals that the mug left undisturbed and undiminished in its contents cooled more slowly. It went from about 72ºC to about 34ºC over the course of an hour. Thirty-eight degrees C, by the way, which is equivalent to about 100ºF, is at the bottom of the temperature range that I call drinkable. The coffee in the other cup cooled more rapidly and in a less regular fashion. This makes sense. Every time I took a drink the volume of the liquid was reduced. Intuitively it seems that a smaller quantity of liquid will cool more rapidly than a larger one. Experientially a drop of water will freeze before a cup of water when both are left in the freezer. More analytically, a larger quantity of liquid has more energy to lose. This combined with the fact that the ratio of volume to surface area for a tall cylinder, a cup filled with coffee, is greater than that of a short cylinder, a cup that is three-quarters empty, gives mathematical support to the idea that the more I drink from the cup the faster it cools. I can see this in the graph; the slope of the line, which represents the rate of cooling, becomes steeper with each succeeding slurp until finally, at 23 minutes, the last swallow is taken and the temperature probe cools rapidly to the temperature of the ceramic cup which is falling toward room temperature, approximately 18ºC.

Eight Cups We have seen that a cup cools faster when we drink from it. If we don't drink from it, though, what's the point? So the next question is what can we do to keep the drink reasonably hot for a reasonable length of time? I decided to test eight different cup arrangements. The first cup was a plastic travel cup. The second was also a travel cup but one made of metal - higher quality, higher price. Is it worth it? The third was a single paper cup while the fourth was a double paper cup. The last four were all ceramic mugs, all of a set. The fifth cup I left alone. The sixth was rinsed with very hot water (pre-warmed) before the liquid was added. The seventh cup received a shot of cream and the final cup was kept on an electric warming device. All received 10 ounces of very hot water (approximately 80ºC, which is 176ºF) at about the same time and were then left undisturbed to cool. Temperatures were monitored with eight temperature probes cabled to a computer, as seen in figure 2.

Figure 2. The eight cups with temperature probes

The cooling curves that resulted are shown in figure 3. As may be seen, not all cups are created equal. Not unsurprisingly, the liquid in the single paper cup (cup #3) cooled more quickly than any other while the metal travel cup (cup #2) was the clear winner in terms of keeping its contents hot. After one hour its contents were 22ºC (nearly 40ºF) hotter than its nearest competitor. The mug seated on the electric heating element (cup #8) was second-warmest after two hours but in the first 15 minutes it made little difference; its contents at that point were in the middle of the pack in terms of temperature.

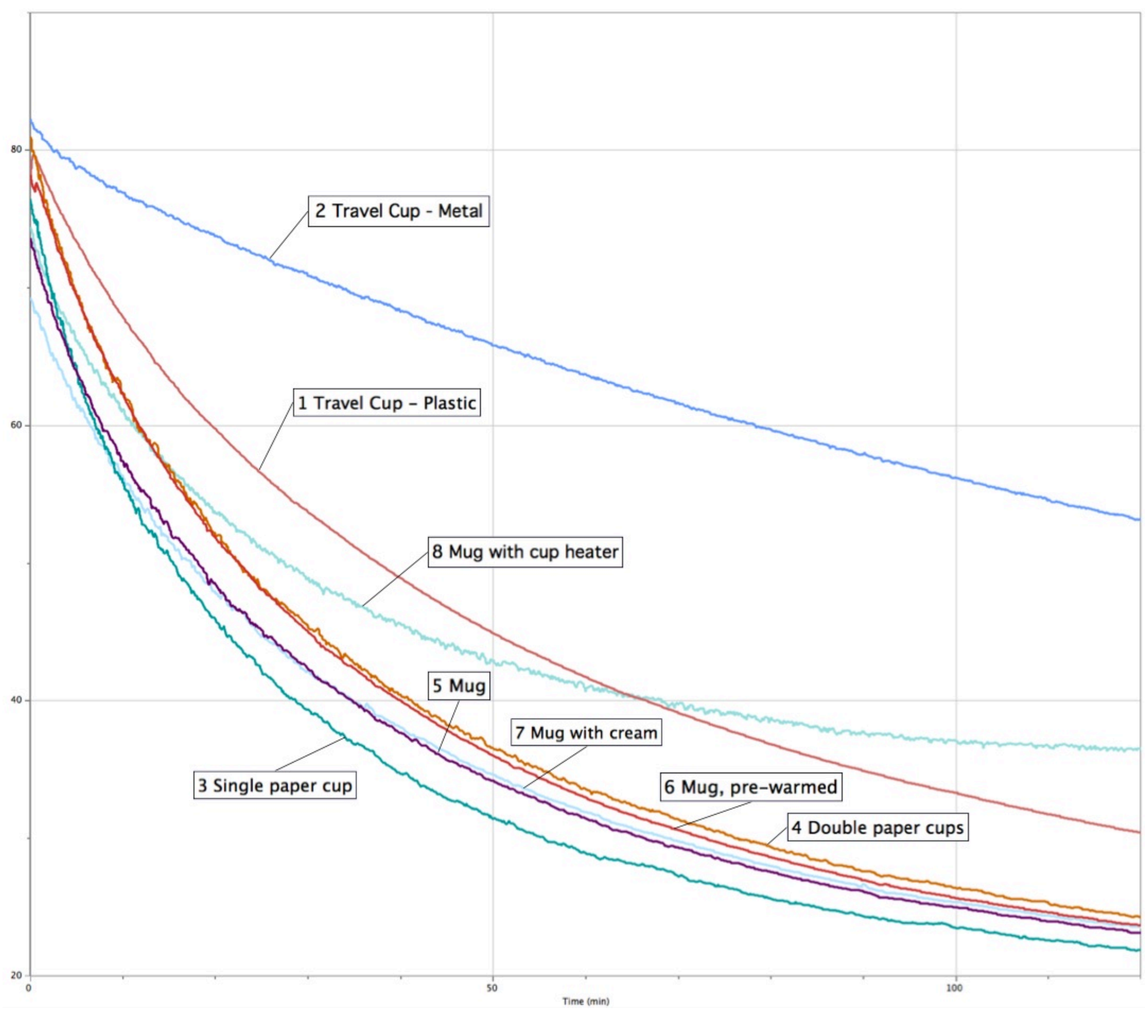

Figure 3. Cooling curves over 120 minutes for all eight cups.

The rates at which temperatures dropped are important to me. I generally finish a cup of coffee within 30 minutes so I am most interested in how temperature changes over this interval. Excluding the two travel cups and the mug on the electric heater, the mean loss in temperature over the first thirty minutes was 31ºC. That means these cups were losing more than a degree every minute. By contrast, the metal travel cup lost only 10ºC over that same interval. The slope of its cooling curve was about one third that of the others. The double paper cup kept its contents 6ºC hotter than did the single paper cup after 30 minutes but is it worth a whole new cup?

At the end of this experience I am left with a host of new questions and investigations I would like to try. The application of mathematics to an everyday situation made it much more interesting. As a math teacher thinking about how I might adapt this for a classroom, I appreciate that this active learning experience begins with choices for the learner (what sort of cup, what sort of liquid and how much), requires kinesthetic involvement, proceeds through measurement to tabular data collection and corresponding graphical representation of the data and leads toward some level of abstraction. I have not discussed modeling the data with equations but that is certainly a reasonable next step. The choice between a simple quadratic and an exponential function might lead to a useful discussion of what happens at extreme values and therefore of the fundamental shapes of the graphs. One fits the situation while the other does not. The fact that this discussion would be rooted in reality and hands-on experience makes it a good way to engender lasting and generative concept development.

Conclusion Coffee will cool regardless of whether the process is analyzed or not. The analysis, though, allows me to attend to an everyday situation with more clarity, precision and, ultimately, enjoyment. The next time a student asks me why they need to know this stuff maybe I'll invite them to discuss it over a cup of coffee or maybe hot chocolate. Will marshmallows make a difference?

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